**Unofficial solutions**

**1a.** See notes

**1bi.** ¬(⋄([p] ∧ [☐p]) ∧ ⋄[¬p])

**1bii.** for all t, u, v, we have R(t, u) and R(u, u) and R(t, v) implies either u = v or R(u, v)

**1ci.** EF(p ∧ q) implies EFp ∧ EFq by definition but EFp ∧ EFq does not imply EF(p ∧ q)

**1cii.** EFp and EFq does not imply EF(p and q) by simple example.

**2ai.** W = {x, y, z}, R = {(x, y), (y, z)} is not transitive.

**2aii.** Let V(p) = {y, z} so the model is M = (W, R, V). Then (M, x) |= ⋄p since (M, y) |= p and R(x, y). But we don’t have (M, y) |= p ∧ ☐ ¬p since p holds at z. So we don’t have (M, x) |= ⋄ (p ∧ ☐ ¬p)

**2bi.** The worlds are { (w, n) | w \in W, n \in {0, 1, 2, ...} }. The relation is R\*((w, n), (w’, n’)) if and only if R(w, w’) and n < n’.  
**2bii.** For any state (w, n), we have ¬ (n < n), so it follows that ¬R\*((w, n), (w, n)). So we have irreflexivity. For any two states (w, n) and (w’, n’), since F is convergent, there exists a state w’’ with R(w, w’’) and R(w’, w’’). Also, n < n + n’ and n’ < n + n’, so we have R\*((w, n), (w’’, n + n’)) and R\*((w’, n’), (w’’, n + n’)). So it is convergent.

**2biii.** Let A be a formula true in all irreflexive and convergent frames. Fix a convergent frame F. The projection F x N -> F which sends (w, n) to w is a surjective p-morphism, so F is a p-morphic image of F x N. From lectures, validity is preserved under p-morphic images, so A is valid in F since it is valid in F x N by assumption.

**2c.** Ga v Gb -> G(a v b) (just unfold definitions. G(a v b) does not imply Ga v Gb, for example consider the model N = ({0, 1, 2, ...}, <) and set V(a) = {0, 2, 4, ...}, V(b) = {1, 3, 5, ..}. Then G(a v b) holds but not Ga v Gb.

**3ai.** False

**3aii.** True

**3aiii.** True

**3bi.** False and True

**3bii.** True and True

**4ai.** True

**4aii.** True

**4bi.** {2, 6, 7, 8, 9, 10, 11, 12}

**4bii.** {3}